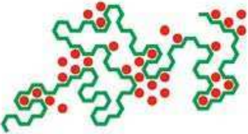
STATISTICS OF LINEAR POLYMERS IN DISORDERED MEDIA



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PREFACE

With the mapping of the partition function graphs of the n-vector magnetic model in the $n \to 0$ limit as the self-avoiding walks, the conformational statistics of linear polymers was clearly understood in early 1970s. Various models of disordered solids, percolation model in particular, were also established by late seventies. Subsequently, investigations on the statistics of linear polymers or of self-avoiding walks in, say, porous medium or disordered lattices were started in early 1980s. Inspite of the brilliant ideas forwarded and extensive studies made, the problem is not yet completely solved in its generality. This intriguing and important problem has remained since a topic of vigorous and active research.

This book intends to offer the readers a first hand and extensive review of the various aspects of the problem, written by the experts in the respective fields. S. M. Bhattacharjee has reviewed the success in dealing with the directed polymers in random medium and has also discussed the problem of unzipping of a pair of directed chains where disorder appears along the chains. A. J. Guttmann has reviewed the series studies of self-avoiding walk statistics for various constrained and random geometries, including the problem of unzipping of the DNA chains. V. Blavats'ka, C. von Ferber, R. Folk and Yu. Holovatch has reviewed extensively the field theoretic and real space renormalization group studies for the problem, including the effects of correlated disorder. D. Dhar and Y. Singh have reviewed most of the exact results for self-avoiding walks on different non-random fractals, including various simplex lattices, employing the real space renormalization group technique. A. Ordemann, M. Porto and H. E. Roman have reviewed the extensive numerical studies on self-avoiding walks on various deterministic and random fractals; percolation clusters in particular. In absence of strict self-avoiding restriction, the analogy of the problem with that of quantum particles in disorder, and the consequent localization of the polymers in random media, have been reviewed by Y. Y. Goldschmidt and Y. Shiferaw. P. Bhattacharyya and A. Chatterjee have reviewed the properties of various optimal and most probable (self-avoiding in general) paths on randomly disordered lattices, including the statistics of the Travelling Salesman Problem on dilute lattices. Finally, G. D. J. Phillies has given an extensive overview of the experimental studies on polymer diffusion in random environments of the solutions.

We earnestly hope, the contents of the book will provide a valuable guide for researchers in statistical physics of polymers and will surely induce further research and advances towards a complete understanding of the problem.

I am grateful to all the contributors for their wonderful contributions and cooperations. I am thankful to Arnab Chatterjee for his help in the compilation of the book.

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Contents

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Self-avoiding walks on deterministic and random fractals: Numerical results

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Numerical techniques for studying statistical properties of self-avoiding random walks (SAWs) on deterministic and random fractal substrates are reviewed. To this end, numerical algorithms are discussed for the generation of SAW configurations of N steps, which are based on Monte Carlo methods and the exact enumeration technique. The advantages and disadvantages of both approaches are highlighted in connection to the different substrates and the statistical quantities of interest. Applying these methods, the scaling behavior (N-dependence) of such static quantities as the total number of SAWs, their end-to-end distance, and the associated probability distribution functions, as well as a possible multifractal behavior, are studied. Scaling forms known for SAWs on regular lattices seem to remain valid also on deterministic and random fractals, while the corresponding relations between the scaling exponents need to be modified in some cases. A prominent example of the latter is the case of percolation at criticality.

1. INTRODUCTION

Self-avoiding walks (SAWs) constitute the simplest, yet non-trivial model for studying the *static* behavior of a linear polymer embedded in a good solvent. Such a polymer is a chain-like array of $N + 1$ monomers rigidly connected to each other, in which the only residual interaction between non-consecutive monomers is a (short-ranged) monomer core repulsion [1–6].

Despite the intrinsic difficulties arising from the emergence of long-range monomer-monomer correlations along such linear chains, many exact results are known (see Chapter 1 by Chakrabarti). If, in addition to this complexity, one aims to study the static behavior of SAWs on substrates displaying self-similarity, one is soon faced with models whose exact solutions become even harder to obtain (possible approaches such as renormalization group analysis and series expansions are discussed in the other chapters of this book). In this more general context, therefore, numerical investigations become a valuable approach to address such issues in a quantitative fashion. Nonetheless, the dif-

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